

# Practice Problems for Mathematics Placement Examination (MPE)

Revised June, 2011

When you come to New Mexico State University, you may be asked to take the Mathematics Placement Examination (MPE). Your initial placement level is based on your Math ACT (or SAT) score and your high school grade point average. The MPE will help to determine in which mathematics course you will be placed. If you are dissatisfied with your initial placement, you may choose to take the MPE and achieve a qualifying score. If you wish to enroll in Math 191 (Calculus I), you *must* take the MPE and achieve a qualifying score.

The problems on this practice exam will help you review your mathematical skills and give you an idea of what is on the test. The actual test is multiple-choice and has 40 problems broken down as follows:

Part I	1-10	Algebra I
Part II	11-20	Algebra II
Part III	21-30	Precalculus Algebra
Part IV	31-40	Trigonometry

These sample problems are grouped in a similar manner. There are more practice problems than are on the actual exam.

Placement Based on the Mathematics Placement Exam		
Placement Exam Score	Placement Level	Math Course Placement
at least 6 on Part I	2	Math 120, 210G
a total of at least 12 on Parts 1 and 2	3	Math 111, 121G, Stat 251, 271
a total of at least 19 on Parts 1, 2 and 3	4	Math 142G, 190G, Math/Hon 275G
at least 6 on each of the 4 parts	5	Math 191G, 235, 278, 279

*Note: An ACT Math score of at least 16 is required to take the Mathematics Placement Exam.*

*A calculator is neither required nor necessary. However you will be allowed to use a non-graphing scientific calculator like the TI-30.*

## PRACTICE PROBLEMS-Review of Basic Skills

The first 15 problems given here are a review of basic arithmetic skills. While not directly tested on the MPE, the knowledge of these skills is essential for the concepts that are tested.

1. Simplify  $52 - 9 \times 4$
2. Simplify  $-5^2 + 10 - 4 \div 2$
3. Find all the positive factors of 24
4. Add  $4\frac{3}{5} + 6\frac{3}{5}$

5. Subtract  $8\frac{2}{3} - 2\frac{6}{7}$
6. Multiply  $2\frac{1}{3} \times 8\frac{1}{4}$
7. Divide  $2\frac{1}{3} \div 5\frac{1}{4}$
8. A carpenter is dividing a board that is  $\frac{3}{4}$  yd long into 9 equal pieces. What is the length of each piece in yards?
9. If  $4\frac{1}{3}$  yd of paper are cut from a roll that is  $18\frac{2}{5}$  yd long, how many yards of paper are left?
10. True or False:  $\frac{5}{12} < \frac{7}{15}$ .
11. Simplify:  $\left(\frac{3}{8}\right)^2 \div \left(\frac{3}{7} + \frac{3}{14}\right)$ .
12. Simplify  $7.3 - 2.1 \times 0.8$
13. What is 7% of 42?
14. What percent of 64 is 72?
15. A store sells 12 oz of Brand A for \$1.20 and 16 oz of Brand B for \$1.50. Which is the more economical buy?

### PRACTICE PROBLEMS FOR PART I - Algebra I

16. Solve for  $x$ :  $\frac{8}{17} = \frac{36}{x}$
17. Solve for  $y$ :  $4y - 17 = -2(5 - y)$
18. If  $z = \frac{2}{3}(x - 12)$  and  $x = 39$ , then what is  $z$ ?
19. Simplify  $\frac{5}{5 + \frac{1}{4}}$
20. Simplify  $|-3 + 4 \times -2|$
21. Rationalize the denominator:  $\frac{\sqrt{2}}{\sqrt{3}}$ .
22. Translate the following statement into an equation:  $S$  equals the quotient of  $r$  and the sum of  $r$  and 8.
23. Collect like terms:  $5a^2bc - 4ab^2c + 3ab^2c^3 + 6a^2bc - ab^2c^3$
24. Simplify and write with positive exponents:  $\frac{(-6x^5y^{-2})^2}{2 \cdot 18y^5x^4}$

25. Combine into a single fraction:  $\frac{5}{x} - \frac{7}{y+1}$ .
26. Find the equation of the line through the points  $(-1, 3)$  and  $(4, -7)$ . Write your answer in slope-intercept form.
27. Find the equation of the line that has slope  $\frac{-4}{3}$  and that goes through the point  $(5, -2)$ .
28. Simplify  $10x + 7(x - z) - z$
29. Simplify  $\frac{14x^3 - 6x^4}{2x^2}$
30. Factor  $x^2 - 9x + 20$ .
31. Factor  $-x^2 + 3x + 28$
32. For the function  $f(x) = -2x^2 + 3x - 5$ , find  $f(2)$  and  $f(-4)$ .
33. The formula for converting Centigrade ( $C$ ) into Fahrenheit ( $F$ ) is given by the formula  $F = \frac{9}{5}C + 32$ . Solve for  $C$  in terms of  $F$ .
34. Thirteen more than eight times a number is the same as two less than eleven times the number. Set up an appropriate equation and solve for the number.
35. A T-shirt company has \$5000 per day in fixed costs and \$4 per T-shirt in production costs. Find the cost function  $C(x)$  that gives the cost of producing  $x$  T-shirts in 3 days.

### PRACTICE PROBLEMS FOR PART II - Algebra II

36. Solve for  $x$  and  $y$ :  $x + y = 10$   
 $y = x + 8$
37. Solve for  $a$  and  $b$ :  $2a - 3b = 5$   
 $-3a + b = 3$
38. Simplify  $\frac{3y^2 + 12y - 36}{y^2 - 16} \cdot \frac{y - 4}{y + 6}$
39. Simplify  $\frac{4}{x^2 - 36} + \frac{2}{x + 6} - \frac{1}{x - 6}$
40. Solve for  $x$ :  $\frac{x}{x - 2} - \frac{3}{x - 1} = 1$
41. Solve for  $a$  in terms of the other variables:  $\frac{1}{A} = \frac{1}{a} + \frac{1}{b}$ .
42. Simplify  $\frac{(-5x^{-2}y^{-2}z)^2}{(10^{-1/3}x^{2/3}yz^{-2/3})^{-3}}$
43. Simplify  $2x\sqrt{12xy^2} - y\sqrt{75x^3}$
44. Simplify  $\sqrt[3]{64x^5y^4z^6}$

45. Write under a single radical and simplify:  $\frac{\sqrt[3]{54a^7d^5}}{\sqrt[3]{2ad}}$
46. Graph the line and label the x and y-intercepts:  $3x - 2y = -6$
47. Find the equation of the line that contains the point  $(-2, -3)$  and that is parallel to the line  $3x + 2y = 6$ . Write your answer in slope-intercept form.
48. Find the equation of the line that contains the point  $(-2, -3)$  and that is perpendicular to the line  $3x + 2y = 6$ . Write your answer in slope-intercept form.
49. Find the vertex and the y and x-intercepts of the function  $f(x) = -x^2 + 4x - 3$ . Graph the function, labelling the vertex and intercepts.
50. The height above ground (in feet) of a toy rocket launched upward from the top of a building is given by  $S(t) = -16t^2 + 96t + 256$ . a) What is the height of the building? b) What is the maximum height attained by the rocket? c) Find the time when the rocket strikes the ground.
51. Find the domain of  $f(x) = \frac{x+2}{x^2-4x-5}$
52. Find the domain of  $g(x) = \sqrt{3-4x}$
53. Solve for  $x$ :  $4^{x-5} = \frac{1}{64}$
54. A colony of 100 termites invades a house. Assuming that the colony doubles in number every 3 weeks and grows continuously, the appropriate mathematical model for the population  $P(t)$  of termites  $t$  weeks in the future is  $P(t) = P_0e^{kt}$ . a) Find the constants  $P_0$  and  $k$ . b) How many termites will there be in 10 weeks? c) When will there be a billion termites?
55. The amount remaining (in grams) of a radioactive substance after  $t$  hours is given by  $A(t) = 100e^{kt}$ . After 12 hours the initial amount has decreased by 7%. a) Solve for the decay constant  $k$ . b) How much of the substance remains after 48 hours? c) What is the half-life of the substance?

### PRACTICE PROBLEMS FOR PART III - Precalculus Algebra

56. The volume of a sphere of radius  $r$  is given by  $V = \frac{4}{3}\pi r^3$ . Solve for  $r$  in terms of  $V$ .
57. Find the distance  $d$  between the points  $(-3, 1)$  and  $(1, 6)$  in the rectangular coordinate system.
58. Solve the inequality  $-5x + 3 \geq 3x - 8$ .
59. Solve the inequalities a)  $|x - 5| \geq 4$  and b)  $|x - 5| < 4$ .
60. If  $f(x) = \frac{4x+3}{x+2}$  and  $g(x) = -2x - 1$ , find  $f(g(x))$ .
61. Factor the following, if possible, over the real numbers:  
 a)  $a^2 - b^2$    b)  $a^2 + b^2$    c)  $a^3 - b^3$    d)  $a^3 + b^3$
62. Express the surface area of a cylinder with a closed bottom and open top in terms of the radius  $r$  and the height  $h$ .

63. Express the surface area of a cube in terms of its side length  $x$ .
64. Graph the parabola  $f(x) = 4x^2 - 3$ . For which values of  $x$  do we have a)  $f(x) < 0$ , b)  $f(x) = 0$ , and c)  $f(x) > 0$ ?
65.  $x^2 - 4x + y^2 + 6y = 3$  is the equation of a circle. Put the equation in standard form and state the center and radius of the circle.
66. Graph the functions  $f(x) = 3^x$  and  $g(x) = 2 \cdot 5^{-x}$  on the same coordinate axes.
67. Graph the function  $f(x) = -5 - 2^{-x}$ .
68. The length  $L$  of a certain rectangle is 4 ft more than three times the width,  $W$ . The perimeter of the rectangle is 300 ft. Set up a system of equations that will allow you to solve for the length and the width, and then find those dimensions.
69. The area of a right triangle is  $266.5 \text{ in}^2$ . The height  $h$  of the triangle is 2 in greater than three times the base  $b$ . Set up a system of equations that will allow you to solve for the base and height, and then find those dimensions.
70. The length  $L$  of a rectangle is twice as much as the width  $W$ . The area of the rectangle is  $30 \text{ ft}^2$ . Set up a system of equations that will allow you to solve for the length and the width, and then find those dimensions.
71. For the function  $f(x) = 2x^2 - 7$ , simplify the expression  $\frac{f(x+h) - f(x)}{h}$ .
72. For the function  $g(x) = \frac{2}{3x-1}$ , simplify the expression  $\frac{g(x+h) - g(x)}{h}$ .
73. Solve the equation  $\log_2(2x-3) = 3$ .
74. Solve the equation  $\log_3 y + 3 \log_3 y^2 = 14$

### PRACTICE PROBLEMS FOR PART IV - Trigonometry

75. Given  $\sin \theta = \frac{1}{x}$  and  $0 \leq \theta \leq \frac{\pi}{2}$ , find the other five trigonometric functions.
76. a) Convert  $315^\circ$  to radians. b) Convert  $\frac{4\pi}{3}$  to degrees.
77. Find the following ranges for the value of  $\theta$  in radians:  
 a)  $0^\circ \leq \theta \leq 90^\circ$  b)  $90^\circ \leq \theta \leq 180^\circ$  c)  $180^\circ \leq \theta \leq 270^\circ$  d)  $270^\circ \leq \theta \leq 360^\circ$
78. a) What is the amplitude of  $f(x) = 4 \sin(2x)$ ? c) What is the period of  $h(x) = 5 \sin(4\pi x)$ ?  
 b) What is the amplitude of  $g(x) = -3 \cos(4x)$ ? d) What is the period of  $k(x) = 6 \tan(2x)$ ?
79. Simplify the expression  $\sin^3 \theta \cot \theta \sec^2 \theta$ .
80. Graph two periods of  $y = \cos(2x)$  where  $x$  is in radians.
81. Find the amplitude, period, horizontal and vertical shifts of  $f(x) = 3 \cos\left(2\left(x - \frac{\pi}{4}\right)\right) + 1$ . Then graph the function for two periods.

82. Complete the following identities:

$$\begin{array}{lll} \text{a) } \sin^2 \theta + \cos^2 \theta = & \text{b) } \tan^2 \theta + 1 = & \text{c) } 1 + \cot^2 \theta = \\ \text{d) } \sin\left(\frac{\pi}{2} - \theta\right) = & \text{e) } \cos\left(\frac{\pi}{2} - \theta\right) = & \text{f) } \sec\left(\frac{\pi}{2} - \theta\right) = \\ \text{g) } \sin 2\theta = & \text{h) } \cos 2\theta = & \end{array}$$

83. a) Find the exact value of  $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ . b) Find the exact value of  $\sin^{-1}\left(-\frac{1}{2}\right)$ .  
c) Find the exact value of  $\tan^{-1}(1)$ .

84. a) Simplify  $\tan\left(\arccos\left(\frac{2}{3}\right)\right)$  b) Express  $\cos(\sin^{-1} x)$  in terms of  $x$  without trig. functions.

85. Find all the solutions of  $2 \sin(x) = 1$  for  $0 \leq x \leq 4\pi$ .

86. Find all the solutions of  $\cos(3x) = -\frac{1}{2}$  for  $0 \leq x \leq 2\pi$ .

87. Solve the following for  $\theta$ , with  $0 \leq \theta < 2\pi$ :  $\cos(2\theta) - \sin \theta = 0$ .

88. Suppose that in a triangle, the side  $a$  is opposite the angle  $\alpha$  and the side  $b$  is opposite the angle  $\beta$ . Use the Law of Sines to solve for  $b$ , given that  $\alpha = 45^\circ$ ,  $\beta = 60^\circ$ , and  $a = \sqrt{6}$ .

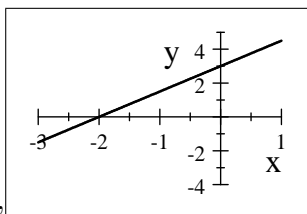
89. Recall that the Law of Cosines states  $b^2 = a^2 + c^2 - 2ac \cos \beta$ , where  $b$  is the side opposite the angle  $\beta$ . Given that  $a = 3$ ,  $b = 5$ , and  $c = 6$ , solve for  $\beta$ . Leave your answer in terms of an inverse trig. function.

90. An observer is standing 2 mi from a very steep mountain. She measures the angle between the ground and the top of the mountain and finds it to be  $45^\circ$ . How high above her ground level is the mountain?

91. The diameter of the base of a cone is 10 ft. Its height is 15 ft Find the angle of inclination of the side of the cone. Leave your answer in terms of an inverse trig. function.

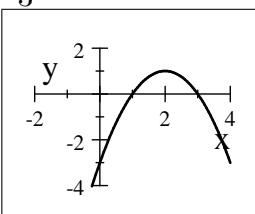
### Practice Problem Answers

- |                                |   |  |                        |                                   |
|--------------------------------|---|--|------------------------|-----------------------------------|
| 1) 16                          | 2) -17                                      | 3) 1, 2, 3, 4, 6, 8, 12, 24                                  | 4) $11\frac{1}{5}$     | 5) $5\frac{17}{21}$               |
| 6) $19\frac{1}{4}$             | 7) $\frac{4}{9}$                            | 8) $\frac{1}{12}$ yd   | 9) $14\frac{1}{15}$ yd | 10) True                          |
| 11) $\frac{32}{7}$             | 12.) 5.62                                   | 13.) 2.94  | 14.) 112.5%            | 15.) Brand B                      |
| 16) 76.5                       | 17) $y = \frac{7}{2}$                       | 18) $z = 18$   | 19) $\frac{20}{21}$    | 20) 11                            |
| 21) $\frac{\sqrt{6}}{3}$       | 22) $S = \frac{r}{r+8}$                     | 23) $11a^2bc - 4ab^2c + 2ab^2c^3$                            | 24) $\frac{2x^6}{y^9}$ | 25) $\frac{-7x + 5y + 5}{xy + x}$ |
| 26) $y = -2x + 1$              | 27) $y = -\frac{4}{3}x + \frac{14}{3}$      | 28) $17x - 8z$   | 29) $7x - 3x^2$        | 30) $(x - 5)(x - 4)$              |
| 31) $-(x - 7)(x + 4)$          | 32) $f(2) = -7, f(-4) = -49$                | 33) $C = \frac{5}{9}(F - 32) = \frac{5}{9}F - \frac{160}{9}$ |                        |                                   |
| 34) $8x + 13 = 11x - 2, x = 5$ | 35) $C(x) = 15000 + 4x$                     | 36) $x = 1, y = 9$   |                        |                                   |
| 37) $a = -2, b = -3$           | 38) $\frac{3(y-2)}{y+4} = \frac{3y-6}{y+4}$ | 39) $\frac{x-14}{(x-6)(x+6)} = \frac{x-14}{x^2-36}$          |                        |                                   |
| 40) $x = 4$                    | 41) $a = \frac{Ab}{b-A}$                    | 42) $\frac{5}{2x^2y}$  |                        |                                   |
| 43) $-xy\sqrt{3x}$             | 44) $4xyz^2\sqrt[3]{x^2y}$                  | 45) $3a^2d\sqrt[3]{d}$                                       |                        |                                   |



46) Intercepts are  $(-2, 0)$  and  $(0, 3)$ ,

47)  $y = -\frac{3}{2}x - 6$       48)  $y = \frac{2}{3}x - \frac{5}{3}$



x-intercepts:  $(1, 0)$  and  $(3, 0)$

y-intercept:  $(0, -3)$

49) vertex:  $(2, 1)$

50) a) 256 ft, b) 400 ft c)  $t = 8$  s    51)  $x \neq -1, x \neq 5$ . Equivalently,  $(-\infty, -1) \cup (-1, 5) \cup (5, \infty)$

52)  $x \leq \frac{3}{4}$ . Equivalently,  $(-\infty, \frac{3}{4}]$ .    53)  $x = 2$

54) a)  $P_0 = 100, k = \frac{\ln 2}{3} = 0.23105$

b)  $P(10) = 100e^{\frac{\ln 2}{3}(10)} = 100 \cdot 2^{\frac{10}{3}} = 1008$  (rounding up).

c)  $t = \frac{21 \ln 10}{\ln 2} = 69.76$  wks.

55) a)  $k = \frac{\ln(0.93)}{12} = -0.00605$

b)  $A(48) = 100e^{4 \ln(0.93)} = 100(0.93)^4 = 74.81$  g

c)  $t = \frac{\ln(0.5)}{k} = \frac{12 \ln(0.5)}{\ln(0.93)} = 114.62$  h

56)  $r = \sqrt[3]{\frac{3V}{4\pi}}$     57)  $d = \sqrt{41}$     58)  $x \leq \frac{11}{8}$ . Equivalently,  $(-\infty, \frac{11}{8}]$ .

59) a)  $x \geq 9$  or  $x \leq 1$ . Equivalently,  $(-\infty, 1] \cup [9, \infty)$     b)  $1 < x < 9$ . Equivalently,  $(1, 9)$ .

60)  $\frac{-8x - 1}{-2x + 1} = \frac{8x + 1}{2x - 1}$

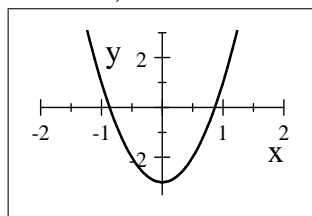
61) a)  $(a - b)(a + b)$

b) Not factorable over the reals.

c)  $(a - b)(a^2 + ab + b^2)$

d)  $(a + b)(a^2 - ab + b^2)$

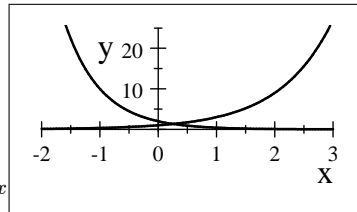
62)  $S = \pi r^2 + 2\pi r h$       63)  $S = 6x^2$



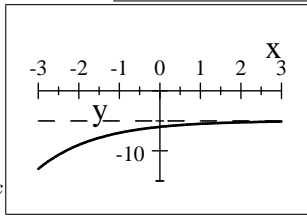
64)  $f(x) = 4x^2 - 3$     a)  $-\frac{\sqrt{3}}{2} < x < \frac{\sqrt{3}}{2}$ . Equivalently,  $(-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2})$

b)  $x = \frac{\sqrt{3}}{2}, x = -\frac{\sqrt{3}}{2}$     c)  $x < -\frac{\sqrt{3}}{2}$  or  $x > \frac{\sqrt{3}}{2}$ . Equivalently,  $(-\infty, -\frac{\sqrt{3}}{2}) \cup (\frac{\sqrt{3}}{2}, \infty)$

65)  $(x - 2)^2 + (y + 3)^2 = 16$ , center is  $(2, -3)$ , radius is 4.



66)  $f(x) = 3^x, g(x) = 2 \cdot 5^{-x}$



67)  $f(x) = -5 - 2^{-x}$

68)  $L = 3W + 4, 2W + 2L = 300, L = \frac{227}{2}$  ft,  $W = \frac{73}{2}$  ft

69)  $\frac{1}{2}bh = 266.5, h = 3b + 2, b = 13$  in,  $h = 41$  in

70)  $L = 2W, LW = 30, L = 2\sqrt{15}$  ft,  $W = \sqrt{15}$  ft

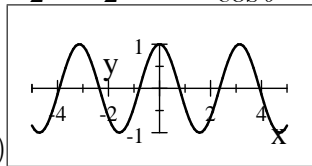
71)  $4x + 2h$  72)  $\frac{-6}{(3x-1)(3x+3h-1)}$  73)  $x = \frac{11}{2}$  74)  $y = 3^{\frac{14}{7}} = 9$

75)  $\cos \theta = \frac{\sqrt{x^2-1}}{x}, \tan \theta = \frac{1}{\sqrt{x^2-1}}, \csc \theta = x, \sec \theta = \frac{x}{\sqrt{x^2-1}}, \cot \theta = \sqrt{x^2-1}$

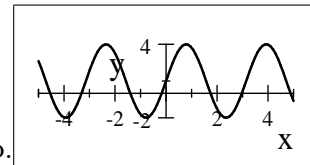
76) a)  $\frac{7\pi}{4}$  b)  $240^\circ$

77) a)  $0 \leq \theta \leq \frac{\pi}{2}$  b)  $\frac{\pi}{2} \leq \theta \leq \pi$  c)  $\pi \leq \theta \leq \frac{3\pi}{2}$  d)  $\frac{3\pi}{2} \leq \theta \leq 2\pi$

78) a) 4 b) 3 c)  $\frac{1}{2}$  d)  $\frac{\pi}{2}$  79)  $\frac{\sin^2 \theta}{\cos \theta} = \sin \theta \tan \theta$



80)  $y = \cos(2x)$



81) Amp = 3, period =  $\pi$ , horiz. shift =  $\frac{\pi}{4}$  right, vert. shift = 1 up.

a) 1 e)  $\sin \theta$

b)  $\sec^2 \theta$  f)  $\csc \theta$

82) c)  $\csc^2 \theta$  g)  $2 \sin \theta \cos \theta$

d)  $\cos \theta$  h)  $\cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$

83) a)  $\frac{\pi}{6}$  b)  $-\frac{\pi}{6}$  c)  $\frac{\pi}{4}$  84) a)  $\frac{\sqrt{5}}{2}$  b)  $\sqrt{1-x^2}$  85)  $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$

86)  $x = \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{8\pi}{9}, \frac{10\pi}{9}, \frac{14\pi}{9}, \frac{16\pi}{9}$  87)  $\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$  88)  $b = 3$

89)  $\beta = \cos^{-1}\left(\frac{-20}{-2 \cdot 18}\right) = \cos^{-1}\left(\frac{5}{9}\right)$  90)  $h = 2$  mi

91)  $\theta = \tan^{-1}(3) = \sin^{-1}\left(\frac{3}{\sqrt{10}}\right) = \cos^{-1}\left(\frac{1}{\sqrt{10}}\right)$